(10) 1. Please derive the formula for the volume of a sphere of radius r.

(15) 2. A sturdy spherical tank of radius 10ft with very thin walls is permanently suspended above a large lake so that the bottom of the tank is just touching the surface of the water. How much work is required to fill this tank with water from the lake, given that water has a density of $32\,\text{lbs}/\text{ft}^3$?

(10) 3. a) Please give a parametrization of the circle of radius 2 centered at the point $(1, 2)$.

b) Use this parametrization to find a formula for the arc length of this circle.

(10) 4. a) The radioactive element Calculonium decays at a rate of 10%/year. If you are given 2kg today, then how long will it take to have half of what you started with?

b) How much will you have after 6 months?

(50) 5. Please evaluate the following integrals, being sure to carefully justify your answer:

a) $\int 4x \sec^2(2x)\, dx$

b) $\int \cos^3(2x) \sin^6(2x)\, dx$

c) $\int \frac{(1-r^2)^{\frac{5}{2}}}{r^8}\,dr$

d) $\int \frac{e^{4t}+2e^{2t}-e^t}{e^{2t}+1}\,dt$

e) $\int_{-2}^{2} x^{-6}\, dx$

(10) 6.

a) Please state the precise definition for a sequence $\{a_n\}_{n=1}^{\infty}$ to converge to a limit L.

b) Use this definition to show that the sequence given by $a_n := \frac{2}{n}$ converges to 0.

(50) 7. Please determine which of the following series converge, being sure to carefully justify your work.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{8\text{Arctan}(n)}{n^2+1}$

b) $\displaystyle\sum_{n=1}^{\infty} (n!)^{\frac{1}{17}}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\ln(n)}$

e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 - 15}{(n^4 + 290)\sqrt{n}}$

(20) 8. a)Please determine the center and radius of convergence of the power series

$$f(x) := \sum_{n=1}^{\infty} \frac{3^{\frac{n}{2}}(x-\pi)^n}{11},$$

being sure to carefully justify your reasoning.

b) What function does it converge to within its region of convergence, and what is the value of the sixth derivative $f^{(6)}(\pi)$?

(20) 9. a) Please find the vector $\vec{v}$ in three dimensional space which represents motion from the point $P = (1, 3, 5)$ to the point $Q = (1, 1, -2)$.

b) What vector $\vec{w}$ can we add to this vector $\vec{v}$ to get the zero vector (which represents no motion)?

c) What is the distance d between P and Q?

d) How is this distance d related to the vector $\vec{v}$?

(0) 10. **(Extra Credit – 10 points):** The Gamma function $\Gamma(x)$ is defined for $x > 0$ by the integral:

$$\Gamma(x) := \int_0^\infty t^{x-1} e^{-t}\, dt.$$

Please evaluate $\Gamma(1)$ and use integration by parts to show $\Gamma(x+1) = x\Gamma(x)$. What is $\Gamma(5)$?

FInal Exam for Math 2260 December 11, 2009

NAME (*please print*): _____

HONOR CODE PLEDGE: _____

SIGNATURE: _____

Please write your answers clearly to all problems, showing all work and carefully justifying your answers. You are not allowed to use any notes or review sheets or calculators during the exam. You have exactly 3 hours to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	10	
2	15	
3	10	
4	10	
5	50	
6	10	
7	50	
8	20	
9	20	
10	0	
Total:	195	