

NAME (*please print*): \_\_\_\_\_

HONOR CODE PLEDGE: \_\_\_\_\_

SIGNATURE: \_\_\_\_\_

Please write your answers clearly to all problems, showing all work and carefully justifying your answers. You are not allowed to use any notes or review sheets or calculators during the exam. You have exactly 50 minutes to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	15	
2	15	
3	40	
4	30	
5	0	
Total:	100	

- (15) 1. Please use the convergence theorems for sequences to prove that the sequence  $\{a_n\}_{n=2}^{\infty}$  with

$$a_n = \left(\frac{n+1}{n-1}\right)^n$$

converges, and determine what value it converges to. Please be sure to carefully justify your reasoning.

- (15) 2. Please prove the “ $p$ -series” rule that

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

converges if  $p > 1$  and diverges if  $0 \leq p \leq 1$ , being sure to carefully justify your reasoning.

- (40) 3. Please determine which of the following series converge, being sure to carefully justify your reasoning.

a) 
$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 7n + 5}$$

b) 
$$\sum_{n=1}^{\infty} \frac{n}{\ln(n^2 + 11)}$$

c) 
$$\sum_{n=1}^{\infty} \frac{3^n (n!)^2}{(3n)!}$$

d) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(2n+1)}$$

- (30) 4. Please determine the regions of absolute and conditional convergence of the Taylor series

$$\sum_{n=1}^{\infty} \frac{2^n}{n} (x-3)^n$$

and identify the function it converges to in this region.

- (0) 5. (Extra Credit – 5 points): Please give a proof of the Pythagorean theorem, carefully justifying your reasoning.