NAME (*please print*): ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

HONOR CODE PLEDGE: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

SIGNATURE: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Please write your answers clearly to all problems, showing all work. You are not allowed to use any notes or review sheets or calculators during the exam. You have exactly 3 hours to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	10	
8	15	
9	20	
10	10	
11	10	
12	0	
13	0	
14	0	
Total:	135	

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(10)   1.   (a) Please carefully state the precise definition of the derivative of a function $f(x)$ at the point $x = a$.

(b) Please use the definition of derivative to compute the instantaneous rate of change of $f(x) = x^2$ at the point $x = a$.

(10)   2.   Please use the limit laws and our elementary limits (but not L'Hopital's rule) to compute the limit
$$\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2},$$
being sure to carefully justify your reasoning.

(10)   3. Please state the sandwich/squeeze theorem and use it to evaluate the limit

$$\lim_{x \to \infty} \frac{\sin(5x)}{x},$$

being sure to carefully justify your reasoning.

(10)   4. Please find the values of x in $[-2, 4]$ where $f(x) = \cot(x) + \lfloor x + \frac{1}{2} \rfloor$ is continuous at x.

(10)   5. Please state the Intermediate Value Theorem and use it to show that the function $f(x) = x^5 - 2$ has a real root, being sure to carefully justify your reasoning.

(20)   6. Please find the derivative of the following functions:

(a) $f(x) = \frac{x^4 + 3x}{\sqrt{x}}$

(b) $f(x) = e^{2x}\cot(3x)$

(c) $f(x) = \ln(\sec^2(x))$

(d) $f(x) = x^{\ln(x)}$

(10) 7. Please find the slope of the tangent line to the curve $y^2 + xy = x^3 + x + y$ at the point $(x, y) = (1, \sqrt{2})$.

(15)   8.   A 13ft ladder is leaning against a house when its base starts to slide away. When the base is 12ft from the house, the base is moving at the rate of 5ft/sec. At what rate is the angle θ between the ground and the ladder changing at that moment?

(20)   9.   Please sketch the graph of the function $f(x) = e^{2/x}$ being sure to label all asymptotes, local and global maxima/minima, inflection points. Also be sure to indicate where the graph is increasing/decreasing, and its concavity in all regions.

(10)   10. A rectangular plot of land will be bounded on one side by a river and on the other three sides
by fence. What is the largest area of land that can enclosed in this way with 800m of fence?

(10)   11.   Please find the area of the bounded region enclosed between the graphs of the functions $f(x) = x^2$ and $g(x) = 2 - x$.

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(0)   12. (**Extra Credit**) Please give a complete proof of the formula

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2},$$

being sure to carefully explain all reasoning.

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(0)   13. (**Extra Credit**) Please evaluate the expression

$$\lim_{x\to\infty} \sum_{n=1}^{\infty} \frac{d}{dx} \int_1^x \frac{\cos(-4\pi e^{1/t})}{2^n}\, dt$$

where "$\sum_{n=1}^{\infty} f(n)$" above means we add up terms $f(n)$ where $n = 1, 2, 3, \cdots$.

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(0)   14. (**Extra Credit**) Please use your knowledge of geometry and the squeeze theorem to show that

$$\lim_{x \to 0^+} \frac{\sin(x)}{x} = 1,$$

being sure to carefully explain all reasoning.