

NAME (*please print*): _____

HONOR CODE PLEDGE: _____

SIGNATURE: _____

Please write your answers clearly to all problems, showing all work. You are not allowed to use any notes or review sheets or calculators during the exam. You have exactly 50 minutes to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

- (20) 1. Please use Newton's method twice to find a good approximation to the root r of $f(x) = x^3 + x + 1$, starting with the closest integer a to r where $f(a) > 0$.

(20) 2. Please evaluate the following limits:

(a) $\lim_{x \rightarrow 0^+} 2 \ln(x) \tan(2x)$

(b) $\lim_{x \rightarrow 0^-} \frac{3}{x^2} + \frac{5}{\sin(x)}$

- (20) 3. Please carefully sketch the graph of a function $f(x)$ satisfying the following properties, being sure to label all local/global extrema, inflection points, concavity, limits and asymptotes:

$$f(x) = \begin{cases} > 0 & \text{if } 1 < x < 2 \text{ or } 5 < x, \\ < 0 & \text{if } x < 1 \text{ or } 2 < x < 5, \\ = 0 & \text{if } x = 1 \text{ or } x = 5, \\ \text{undefined} & \text{if } x = 2. \end{cases}$$

$$f'(x) = \begin{cases} > 0 & \text{if } -2 < x < 2 \text{ or } 2 < x, \\ < 0 & \text{if } x < -2, \\ = 0 & \text{if } x = -2, \\ \text{undefined} & \text{if } x = 2. \end{cases}$$

$$f''(x) = \begin{cases} > 0 & \text{if } -5 < x < 0 \text{ or } 3 < x, \\ < 0 & \text{if } x < -5 \text{ or } 0 < x < 2 \text{ or } 2 < x < 3, \\ = 0 & \text{if } x = -5 \text{ or } x = 3, \\ \text{undefined} & \text{if } x = 2. \end{cases}$$

$$\lim_{x \rightarrow \infty} = \infty, \quad \lim_{x \rightarrow -\infty} = -3, \quad \lim_{x \rightarrow 2^+} = -\infty, \quad \lim_{x \rightarrow 2^-} = 3,$$

$$f(0) = f(3) = -3, \quad f(-2) = -7, \quad f(-5) = -5.$$

- (20) 4. Please find the volume of the largest right circular cone that can be inscribed in a sphere of radius 5.

(20) 5. (a) Please evaluate $\int x^5 + xe^{(3x^2)} dx$.

(b) Please find the unique solution $f(x)$ of the differential equation

$$\frac{df}{dx} = \frac{5x}{2x^2 + 1},$$

where $f(1) = 3$.