Final Project: Cyclotomic numbers and Fermat's last theorem

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Abstract

In class we have discussed the ring $\mathbb{Z}[i]$ where $i = \sqrt{-1}$. One can also study more general number systems involving roots of 1 called the *cyclotomic integers*, which look like $\mathbb{Z}[\zeta_p]$ where ζ_p is a p^{th} root of 1 (and p is an odd prime). These number systems play a central role in much of number theory, and their arithmetic was used to prove many cases of Fermat's last theorem. Your project should describe the cyclotomic integers, and explain the proof of Fermat's last theorem in the case where the $\mathbb{Z}(\zeta_p)$ has unique prime factorization. **[4-5]**

The following is a rough outline which may be useful in thinking about/organizing your project. A very good reference for this project is [1, Chapters 4–6], though it may be interesting to compare that perspective with some of the lectures in [3]. There are also some comments in [2, Chapter 17, §1,2,4,8,11]. If you have any questions about your project and/or readings, feel free to let me know, and we can setup a time to talk about it. Have Fun! =)

1. **Integers and factoring in the cyclotomic integers** $\mathbb{Z}[\zeta_p]$
2. **Lamé's "proof" of Fermat's last theorem**
3. **The proof for n=3**
4. **Unique prime ideal factorization**
5. **Computing the class number**
6. **Relation to the Bernoulli numbers**
7. **Other proofs of Fermat's Last theorem**
8. **Comments about Wiles's proof**

References

[1] Harold M. Edwards. *Fermat's last theorem*, volume 50 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996. A genetic introduction to algebraic number theory, Corrected reprint of the 1977 original.

[2] Kenneth Ireland and Michael Rosen. *A classical introduction to modern number theory*, volume 84 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1990.

[3] Paulo Ribenboim. *13 lectures on Fermat's last theorem*. Springer-Verlag, New York, 1979.

