

NAME (*please print*): _____

Honor Code Reaffirmation: _____

“I have adhered to the UGA Honor Code in completing this assignment.”

SIGNATURE: _____

Please write your answers clearly to all problems, showing all work carefully explaining your answers. You are not allowed to use any notes, review sheets or calculators during the exam. You have exactly 75 minutes to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	20	
2	40	
3	20	
4	20	
Total:	100	

(20) 1. a) Given a function $f(x)$ defined on the interval $[a, b]$, please precisely define what it means for $f(x)$ to be increasing on $[a, b]$.

b) Please precisely state Rolle's theorem.

c) Please use Rolle's theorem to show that the function $f(x) = x(x - 1) \sin(x^{11} + 17)$ has $f'(a) = 0$ for some real number a . Be sure to show all work and carefully justify your answers.

- (40) 2. Suppose we have a function $f(x)$ defined for all real numbers except $x = 1$, whose derivatives are given by

$$f'(x) = \frac{x-5}{(x-1)^3} \quad f''(x) = \frac{2(7-x)}{(x-1)^4}.$$

Also suppose that $f(0) = 2$ and $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = -1$. Please be sure to show all work and carefully justify your answers.

a) Please determine the critical points of $f(x)$.

b) Please determine on which intervals the function $f(x)$ is increasing and decreasing.

c) Please determine the local extrema of $f(x)$, and state which kinds of extrema (max/min) occur at which points.

d) Please determine the intervals on which $f(x)$ is concave up and concave down.

e) Please determine the inflection points of $f(x)$.

f) Please determine the local extrema of $f'(x)$, and indicate which kinds of extrema occur at which points.

g) What is the largest slope that occurs for a tangent line to the graph $y = f(x)$?

h) Sketch the graph of $y = f(x)$, using all known information.

- (20) 3. Suppose that a farmer wants to keep as many wild pigs as possible in a rectangular fenced region along a very long pre-existing straight wall, which will form the fourth side of the wild pig habitat. How much space can he arrange to have for his wild pig farm if the farmer only has 200 yards of fence to use to build the habitat? Please be sure to carefully show all work.

- (20) 4. Suppose that your friend purchased a spherical balloon full of helium and has measured its radius as being 1 foot, but he used a slightly bent ruler which is accurate to only 0.1 ft. About how much error will be in his computation of the volume of the helium in the balloon? Please be sure to carefully show all work.