

NAME (*please print*): _____

Honor Code Reaffirmation: _____

“I have adhered to the UGA Honor Code in completing this assignment.”

SIGNATURE: _____

Please write your answers clearly to all problems, showing all work carefully explaining your answers. You are not allowed to use any notes, review sheets or calculators during the exam. You have exactly 75 minutes to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	30	
2	30	
3	30	
4	30	
5	30	
Total:	150	

(30) 1. Please evaluate the following limits, being sure to show all work:

(a) $\lim_{\theta \rightarrow 0} \frac{\theta + 1}{\theta - 1}$

(b) $\lim_{x \rightarrow 0} \frac{x^2}{x^3 - x}$

(c) $\lim_{z \rightarrow 0} \frac{\tan(5z)}{\cos(2z)}$

(d) $\lim_{z \rightarrow 0} \frac{\tan(5z)}{\sin(2z)}$

(e) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x - 2}$

(30) 2. (a) Please precisely state the (limit) definition of the derivative $f'(x)$ of a function $f(x)$ at x .

(b) Please use this definition to prove that the derivative of $f(x) = \frac{1}{x^2}$ is given by $f'(x) = \frac{-2}{x^3}$.

(c) Please use your results above to compute the equation of the tangent line to the graph of $f(x) = \frac{1}{x^2}$ when $x = 1$.

(30) 3. (a) Please precisely define what it means for a function $f(x)$ to be continuous at $x = c$.

(b) Please precisely define what it means for a function $f(x)$ to be continuous on the interval (a, b) .

(c) Please draw the graph of a function $f(x)$ that has the following properties:

- $f(x)$ is defined for all real numbers except when $2 \leq x \leq 3$.
- $f(x)$ is continuous on the intervals $(-\infty, 1)$ and $(3, \infty)$.
- $f(x)$ is *not* differentiable when when $x = 4$.
- $f'(0) = 1$ and $f(0) = -2$.

(30) 4. Please evaluate the following one sided limits, being sure to show all work:

$$(a) \lim_{x \rightarrow 0^-} \frac{|x|}{|x - 2|}$$

$$(b) \lim_{x \rightarrow 0^+} \frac{|x|}{|x - 2|}$$

$$(c) \lim_{x \rightarrow 2^-} \frac{|x|}{|x - 2|}$$

$$(d) \lim_{x \rightarrow 2^+} \frac{|x|}{|x - 2|}$$

In the following two problems, the function $[x]$ is the “greatest integer” function we discussed in class, that is defined to be the largest integer $\leq x$.

$$(e) \lim_{x \rightarrow 0^-} \frac{[x]}{x}$$

$$(f) \lim_{x \rightarrow 0^+} \frac{[x]}{x}$$

- (30) 5.
(a) Please precisely state the Intermediate Value Theorem.

(b) Please use the Intermediate Value Theorem to show that the function $f(x) = 2 \sin(x) - x + 1$ has a root in the interval $[0, \pi]$.