In-Class Exam for Hanke – Math 2200 – 8:00am October 11, 2011

NAME (*please print*): Answer Key

Honor Code Reaffirmation: Test #2

"I have adhered to the UGA Honor Code in completing this assignment."

SIGNATURE: Jonathan Hanke

Please write your answers clearly to all problems, showing all work carefully explaining your answers. You are not allowed to use any notes, review sheets or calculators during the exam. You have exactly 75 minutes to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

 Name: ____________________

(20) 1. Please differentiate each of the following functions (with respect to x), being sure to show all work:

a) $x^2 + 3\cos(x)$

$$\Rightarrow f'(x) = 2x + 3(-\sin(x))$$

$$= \boxed{2x - 3\sin(x)}$$

b) $e^{2x}\cot(5x)$

$$\Rightarrow f'(x) = e^{2x} \cdot [-\csc^2(5x) \cdot 5] + \cot(5x) \cdot [e^{2x} \cdot 2]$$

$$= e^{2x} \cdot [2 \cdot \cot(5x) - 5\csc^2(5x)].$$

c) $\frac{\pi}{\sqrt[3]{2x^5+9}} = \pi \cdot (2x^5+9)^{1/3}$

$$\Rightarrow f'(x) = \frac{\pi}{3} \cdot (2x^5+9)^{-2/3} \cdot 10x^4$$

$$= \boxed{\frac{10\pi}{3} \cdot \frac{x^4}{(2x^5+9)^{2/3}}}$$

d) $\frac{\ln(\sec^2(x))}{x} = \frac{1}{x} \cdot 2\ln(\sec(x)) = \frac{-2\ln(\cos(x))}{x}$

$$\Rightarrow f'(x) = \frac{x \cdot (-2 \cdot \frac{1}{\cos(x)} \cdot \sin(x)) - -2\ln(\cos(x)) \cdot 1}{x^2}$$

$$= \boxed{\frac{2\ln(\cos(x)) - 2x \cdot \tan(x)}{x^2}}$$

(20) 2. Please find the equation of the tangent line to the graph $(x+y)^3 = x^3 + y^3$ at the point $P = (-1, 1)$, being sure to show all work.

Find slope $= \frac{dy}{dx}$ at $P=(-1,1)$ by implicit differentiation:

$$(x+y)^3 = x^3+y^3 \overset{\frac{d}{dx}}{\Longrightarrow} 3(x+y)^2\cdot\left(\overset{1}{\cancel{\frac{dx}{dx}}} + \frac{dy}{dx}\right) = 3x^2\overset{1}{\cancel{\frac{dx}{dx}}} + 3y^2\frac{dy}{dx}$$

$$\Rightarrow 3(x+y)^2\left(1+\frac{dy}{dx}\right) = 3x^2+3y^2\frac{dy}{dx}$$

$$\Rightarrow 3(x+y)^2 + 3(x+y)^2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$$

$$\Rightarrow 3(x+y)^2 - 3x^2 = 3y^2\frac{dy}{dx} - 3(x+y)^2\frac{dy}{dx}$$

$$\Rightarrow 3(x+y)^2 - 3x^2 = \left(3y^2 - 3(x+y)^2\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(x+y)^2 - 3x^2}{3y^2 - 3(x+y)^2}$$

$$\overset{(x,y)=(-1,1)}{\Longrightarrow} \frac{dy}{dx} = \frac{3(\cancel{-1+1})^2 - 3(-1)^2}{3\cdot 1^2 - \cancel{3(-1+1)^2}} = \frac{-3}{3} = -1$$

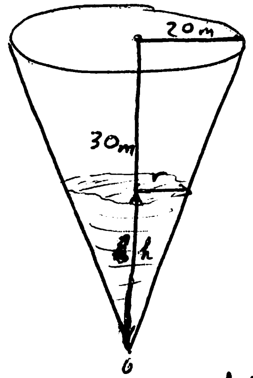
Point-Slope Formula $\Rightarrow y - 1 = -1(x + +1)$

$$\Rightarrow \boxed{y-1 = -x-1}$$

(20) 3. A conical water tower (with point pointing down) has a top radius of 20 meters and height of 30 meters. From the time it is completely filled, the water starts to run out of the bottom at the rate of 1 cubic meter per day. How quickly is the water level falling when the water level is at 10 meters from the bottom of the tank?

(Please draw a reasonable picture labelling all important quantities with either variables or constants, and be sure to show all necessary work to justify your conclusion!)

Picture:



$\downarrow \frac{dV}{dt} = -1 \ m^3/day$

Want: $-\frac{dh}{dt}$ when $h = 10m$.

Relationship between r & h by similar triangles:

$$\frac{r}{h} = \frac{20m}{30m} \Rightarrow \boxed{r = \frac{2}{3}h}$$

Relationship between V and h:

$$V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 \cdot h$$

$$\Rightarrow \boxed{V = \frac{4\pi}{27}h^3}$$

Differentiate and Solve:

$$\Rightarrow \frac{dV}{dt} = \frac{4\pi}{27} \cdot 3h^2 \frac{dh}{dt} = \frac{4\pi}{9}h \cdot \frac{dh}{dt}$$

(when $h=10$)

$$\Rightarrow -1 = \frac{4\pi}{9} \cdot 10 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = \frac{-9}{40\pi} \ m/day$$

Conclusion: The water level is falling at $\frac{9}{40\pi}$ m/day when $h = 10m$.

(20) 4. Suppose the function $f(x) = e^x \sin(x)$ is defined on the closed interval $[0, \pi]$. In the following problem you will be asked to find the absolute extrema of $f(x)$.

(Please be sure to carefully justify all reasoning, and show all work!)

(a) What are the critical numbers for $f(x)$? ← (When $f'(x) = 0$ or DNE in $[0, \pi]$.)

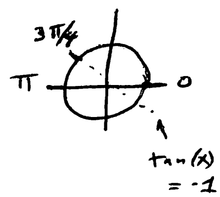
$$f'(x) = e^x \cdot \cos(x) + \sin(x) \cdot e^x = e^x(\cos(x) + \sin(x))$$

a) $f'(x) = 0 \Rightarrow e^x = 0$ or $\cos(x) = -\sin(x)$

↑ (Never since $e^x > 0$ always!)

$\Rightarrow \tan(x) = -1$

$\Rightarrow \boxed{x = \frac{3\pi}{4}}$



b) $f'(x) = \text{DNE} \Rightarrow e^x(\cos(x) + \sin(x)) = \text{DNE}$

(Never since all functions here are cts everywhere!)

$\Rightarrow \boxed{\text{Critical \#s: } x = \frac{3\pi}{4}}$

(b) What are the absolute extrema of $f(x)$? ← (Check endpts and critical #s)

$f(0) = e^0 \cdot \sin(0) = 0$

$f(\pi) = e^\pi \cdot \sin(\pi) = 0$

$f(3\pi/4) = e^{3\pi/4} \cdot \sin(3\pi/4) = e^{3\pi/4} \cdot \frac{\sqrt{2}}{2} > 0$

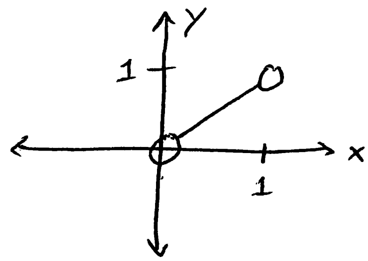
$\Rightarrow$ $\boxed{\begin{array}{l} \text{Global Max} = e^{3\pi/4} \cdot \frac{\sqrt{2}}{2} \\ \text{Global Min} = 0 \end{array}}$

(20) 5. a) Please precisely state the theorem from class that ensures that a function has a global maximum and minimum.

Extreme Value Theorem: Suppose $f(x)$ is a continuous function on a closed interval $[a, b]$. Then $f(x)$ attains a global maximum and a global minimum on $[a, b]$.

b) Please give an example of a function that does not have a global maximum, and carefully explain why your example does not satisfy the conditions of this theorem.

Example: $f(x) = x$ on $(0, 1)$.



This function has no global maximum ~~but~~ because its values lie in $(0,1)$ ← (This is the range of $f(x)$.)

This function does not satisfy the conditions of the Extreme Value Theorem since it is not defined on a closed interval.