Final Project: Primes in arithmetic progressions

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February 14, 2005

Abstract

In class we will show that there are infinitely many prime numbers, and infinitely many of the form $4k+1$ and $4k+3$. By a remarkable analytic method, Dirichlet showed that there are infinitely many primes of the form $ak+b$ so long as the integers a and b are relatively prime. Your paper should explain his idea and proof, and work out a few specific examples of this remarkable theorem in detail. **[3.5-4.5]**

The following is a rough outline which may be useful in thinking about/organizing your project. Good general references are [4, Chapter VII], [3, Chapter 16, §1–5], [1, Chapters 1 and 4]. It is also discussed briefly in [2, pp365–370] and in [5, §11.4]. If you have any questions about your project and/or readings, feel free to let me know, and we can setup a time to talk about it. Have Fun! =)

1. **Infinitely many primes using the Riemann zeta function** $\zeta(s)$
2. **Infinitely many primes of the form** $4k+1$ **and** $4k+3$
3. **Dirichlet Characters and** L**-functions**

   It may be easier to consider only characters with prime modulus first, and then see how a composite modulus can be understood using its factors.
4. **General facts about convergence of Dirichlet series**
5. **The pole at** $s=1$ **from** $\sum_p \frac{\chi(p)}{p}$ **for the trivial character**
6. **Non-vanishing of** $L(1,\chi)$ **for non-trivial characters**

References

[1] Harold Davenport. *Multiplicative number theory*, volume 74 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2000.

[2] Harold M. Edwards. *Fermat's last theorem*, volume 50 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996. A genetic introduction to algebraic number theory, Corrected reprint of the 1977 original.

[3] Kenneth Ireland and Michael Rosen. *A classical introduction to modern number theory*, volume 84 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1990.

[4] Anthony W. Knapp. *Elliptic curves*, volume 40 of *Mathematical Notes*. Princeton University Press, Princeton, NJ, 1992.

[5] Jeffrey Stopple. *A primer of analytic number theory*. Cambridge University Press, Cambridge, 2003. From Pythagoras to Riemann.