

Final Project: Geometry and Sums of 4 squares (Minkowski's theorem)

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February 14, 2005

Abstract

One of the most amazing insights into the “geometry” of numbers was offered by a brilliant young mathematician named Hermann Minkowski, who showed that the distribution of lattice points in a symmetric region is related to the area/volume of the region. This theorem is quite easy to state and prove, but has many surprising applications in number theory. One of these is to show that every natural number is a sum of 4 squares. Your project should explain the theorem, and show how it can be used to answer some questions related to which numbers are sums of squares. [2.5-4]

The following is a rough outline which may be useful in thinking about/organizing your project. Good general references are [2, Chapter 22], [4], [3, §3.9], and [1, pp98–102]. If you have any questions about your project and/or readings, feel free to let me know, and we can setup a time to talk about it. Have Fun! =)

1. **Minkowski's Theorem**
2. **Some primes as a sum of 2 squares**
3. **All numbers as a sum of 4 squares**
4. **Local-Global principle for conics**
5. **Other applications**

References

- [1] J. W. S. Cassels. *An introduction to the geometry of numbers*. Classics in Mathematics. Springer-Verlag, Berlin, 1997. Corrected reprint of the 1971 edition.
- [2] Jay R. Goldman. *The queen of mathematics*. A K Peters Ltd., Wellesley, MA, 1998. A historically motivated guide to number theory.
- [3] G. H. Hardy and E. M. Wright. *An introduction to the theory of numbers*. The Clarendon Press Oxford University Press, New York, fifth edition, 1979.
- [4] C. D. Olds, Anneli Lax, and Giuliana P. Davidoff. *The geometry of numbers*, volume 41 of *Anneli Lax New Mathematical Library*. Mathematical Association of America, Washington, DC, 2000. Appendix I by Peter D. Lax.

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