Final Project: Solving $x^3 + y^3 = z^3$ and $x^4 + y^4 = z^4$

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Abstract

While there are infinitely many integer solutions to the equation $x^2 + y^2 = z^2$, there are no solutions of the equation $x^n + y^n = z^n$ for any integer power $n > 2$ where one of x, y, z are not zero. This is called Fermat's Last theorem, even though his supposed proof has never been found. Very recently, a proof of this has been found using elliptic curves and modular forms. However, Fermat was able to prove the cases $n = 3$ and $n = 4$ by his method of "infinite descent". Your project should explain Fermat's proof for $n = 3$ and 4, and its relationship to descent on elliptic curves. **[3.5-5]**

The following is a rough outline which may be useful in thinking about/organizing your project. Good references are [1, pp168–170], [2, pp1–14], [4, Chapter 17, §1, §2, §8], [3, §13.1–4]. The more lofty connection to "descent" on an elliptic curve is discussed in [5, pp80–84 and pp119–122 for solving $x^4 + y^4 = z^2$] and very briefly in [**?**, pp17–18]. If you have any questions about your project and/or readings, feel free to let me know, and we can setup a time to talk about it. Have Fun! =)

1. **Pythagorean triples and** $x^2 + y^2 = z^2$
2. **Fermat's proof of** $x^3 + y^3 = z^3$
3. **Connection to Unique factorization in** $\mathbb{Q}(\omega)$
4. **Euler's proof of** $x^4 + y^4 = z^2$
5. **The associated elliptic curves**
6. **Connection to 2-descent on these elliptic curves**

References

[1] H. Davenport. *The higher arithmetic*. Cambridge University Press, Cambridge, seventh edition, 1999. An introduction to the theory of numbers, Chapter VIII by J. H. Davenport.

[2] Harold M. Edwards. *Fermat's last theorem*, volume 50 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996. A genetic introduction to algebraic number theory, Corrected reprint of the 1977 original.

[3] G. H. Hardy and E. M. Wright. *An introduction to the theory of numbers*. The Clarendon Press Oxford University Press, New York, fifth edition, 1979.

[4] Kenneth Ireland and Michael Rosen. *A classical introduction to modern number theory*, volume 84 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1990.

[5] Anthony W. Knapp. *Elliptic curves*, volume 40 of *Mathematical Notes*. Princeton University Press, Princeton, NJ, 1992.