# Final Project: Dirichlet's class number formula

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**Abstract**

Because of it's connection to unique prime factorization (described above), it is important to be able to compute the class number $h$ of a number system. By a remarkable analytic method, Dirichlet was able to give an explicit formula for the class number in certain cases using a variant of the Riemann zeta function $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$. Your project should describe how this is done, and describe how one can use the to compute the class number of the integers in any quadratic number field $\mathbb{Q}(\sqrt{D})$, and perhaps for other number systems in $\mathbb{Q}(\sqrt{D})$. **[3-4]**

The following is a rough outline which may be useful in thinking about/organizing your project. Good general references are [2, Ch 6], [3, Chapters 7 and 9], and [1, especially Chapter 10]. It is also discussed in [4, Chapters 11 and 13.1–2]. If you have any questions about your project and/or readings, feel free to let me know, and we can setup a time to talk about it. Have Fun! =)

1. **Class numbers and ideals**
2. **The Riemann zeta function $\zeta(s)$ and its Euler product**
3. **The zeta function $\zeta_{\mathbb{Q}(\sqrt{D})}(s)$ and its Euler product**
4. **Relating $\zeta(s)$ and $\zeta_{\mathbb{Q}(\sqrt{D})}(s)$**
5. **Poles and residues of $\zeta(s)$ and $\zeta_{\mathbb{Q}(\sqrt{D})}(s)$**
6. **The class number formula**
7. **Computing the class number and/or the fundamental unit**

## References

[1] Harvey Cohn. *Advanced number theory*. Dover Publications Inc., New York, 1980. Reprint of *A second course in number theory*, 1962, Dover Books on Advanced Mathematics.

[2] Harold Davenport. *Multiplicative number theory*, volume 74 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2000.

[3] Harold M. Edwards. *Fermat's last theorem*, volume 50 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996. A genetic introduction to algebraic number theory, Corrected reprint of the 1977 original.

[4] Jeffrey Stopple. *A primer of analytic number theory*. Cambridge University Press, Cambridge, 2003. From Pythagoras to Riemann.