NAME (*please print*): ────────────────────────────

Honor Code Reaffirmation: ────────────────────────────
    "I have adhered to the UGA Honor Code in completing this assignment."

SIGNATURE: ────────────────────────────

Please write your answers clearly to all problems, showing all work carefully explaining your answers. You are not allowed to use any notes, review sheets or calculators during the exam. You have exactly 3 hours to complete the exam. Good Luck!

Problem Number	Possible Points	Points Earned:
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	10	
11	20	
12	20	
13	20	
Total:	250	

(20)   1. Please evaluate the following limits:
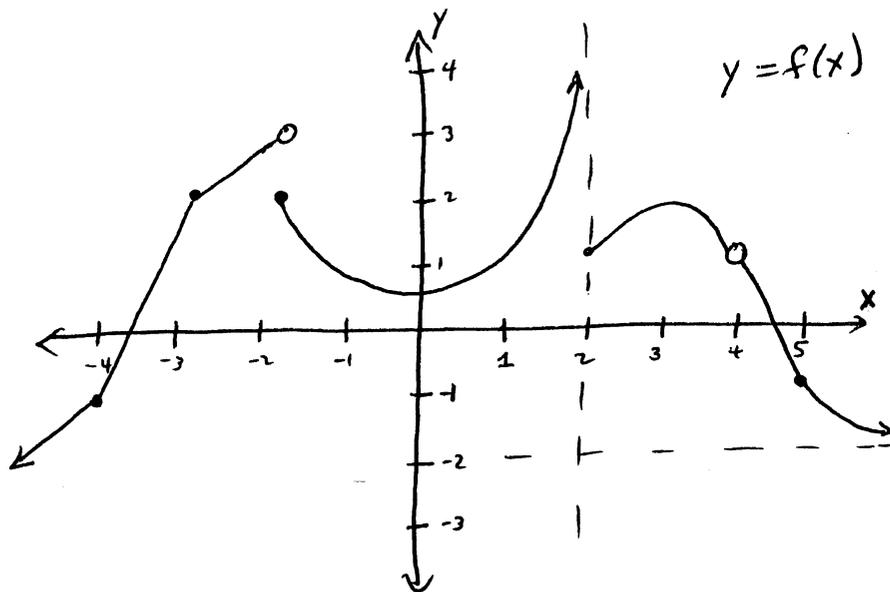
a) $\lim\limits_{z \to 1} \dfrac{z}{z+1}$

b) $\lim\limits_{t \to 0} \dfrac{2t^3}{t^4 - t^3}$

c) $\lim\limits_{r \to -\infty} \dfrac{3r^3 + 5r^2 + 11}{2r^4 + e^r}$

Please use your limit rules to compute and carefully justify the following limit:

d) $\lim\limits_{x \to 0} \dfrac{5\sin(x)\cos(2x)}{\tan(3x)}$

(20)   2. Please use the graph

$y = f(x)$

to evaluate the following limits:

a) $\lim\limits_{x \to -2^+} f(x)$

b) $\lim\limits_{x \to -2^-} f(x)$

c) $\lim\limits_{x \to \infty} f(x)$

d) $\lim\limits_{x \to -\infty} f(x)$

e) $\lim\limits_{x \to 4} f(x)$

f) At which real numbers c does $\lim\limits_{x \to c^-} f(x)$ exist?

(20)   3.   a) Please precisely state the definition of the derivative $f'(x)$ of a function $f(x)$ at x.

b) Please use the definition of derivative (i.e. no derivative rules!) and your knowledge of limits to evaluate the derivative $f'(x)$ of the function $f(x) = \sqrt{x}$, being sure to carefully justify your work. (Feel free to check your answer using the usual derivative rules, but no credit is given for this.)

(20)   4. For each of the following graphs, please determine the intervals where $f'(x) > 0$, and also the intervals where $f''(x) > 0$.
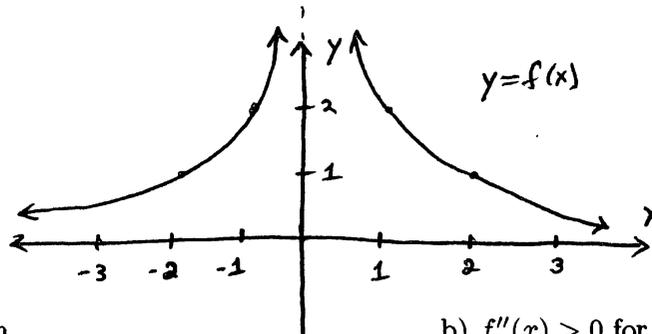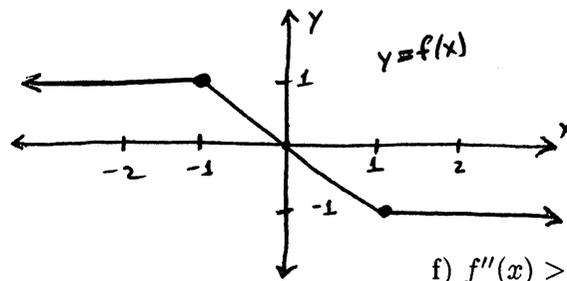

a) $f'(x) > 0$ for x in   b) $f''(x) > 0$ for x in
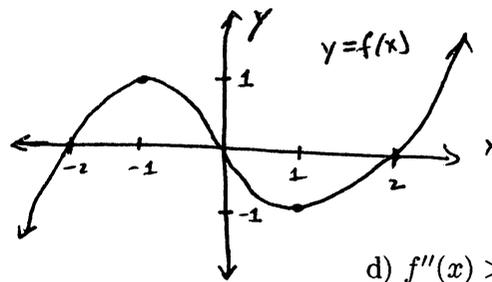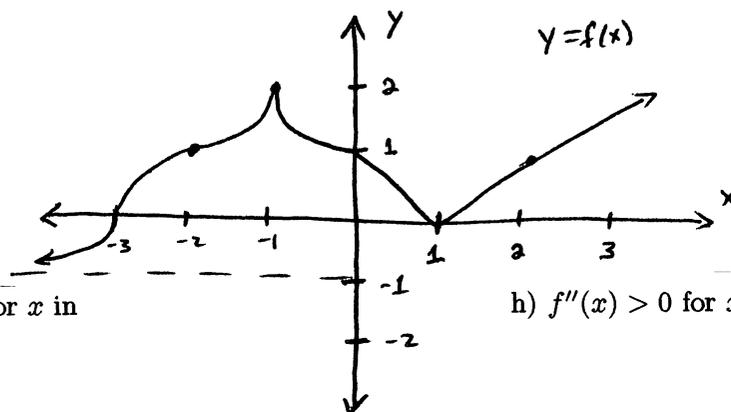

c) $f'(x) > 0$ for x in   d) $f''(x) > 0$ for x in

e) $f'(x) > 0$ for x in   f) $f''(x) > 0$ for x in

g) $f'(x) > 0$ for x in   h) $f''(x) > 0$ for x in

(20)   5. Please determine whether each of the following statements is either **True** (i.e. always True) or **False** (i.e. sometimes False).

a) Every function $f(x)$ that is continuous at $x = c$ is differentiable at $x = c$.

b) Every function $f(x)$ that is differentiable at $x = c$ is continuous at $x = c$.

Suppose that the function $g(x)$ is defined by the two rules: $g(x) = \cos(x)$ when $x \geq 0$ and $g(x) = x^2 + 1$ when $x < 0$.

c) $g(x)$ is continuous at $x = 0$.

d) $g(x)$ is differentiable at $x = 0$.

e) Every continuous function on an open interval (a, b) attains both a maximum and a minimum value on (a, b).

(20)   6.   Please use your derivative rules to evaluate the derivatives $f'(x)$ for each of the following functions $f(x)$.

a) $f(x) = x^3$

b) $f(x) = \ln(x) + e^x$

c) $f(x) = \cos(\sin(3x^2))$

d) $f(x) = \sec(x) \cdot \tan(x)$

e) $f(x) = \ln\left(\frac{17}{\sqrt{x}}\right)$

f) $f(x) = \frac{\csc(x)}{x^2}$

(20) 7. a) Please find the slope of the tangent line to the graph $x^3 + y^3 = 9$ at the point (x, y).

b) Please find the equation of the tangent line to the graph at the point $(x, y) = (2, 1)$.

(20)   8. Suppose you decide to keep wild Smurfs as a hobby, and decide to build a rectangular pen for them using 50 feet of Smurf-proof fencing and using a pre-existing wall as one side of your pen. What is the maximum amount of space you can arrange to have for keeping Smurfs? Please be sure to carefully justify your answer and show all work!

(20)   9. Suppose that you are making a dramatic movie that involves the sun setting, which you simulate by lowering a bright light from the ceiling of your 30 foot tall movie studio to the floor directly below it. You are illuminating a 6 foot tall actress who standing 8 feet away from the spot directly below the bright light. If the light is being lowered at a constant rate of 2ft/sec, how quickly is the actress's shadow lengthening when the light is 12 feet above the ground? Please be sure to carefully justify your answer, and show all work!

(10)   10. Use differentials to estimate the error in the volume of a cylinder whose height is known to be exactly 2 feet, but whose radius is approximately 3 feet, with an error of about 0.1ft.

(20) 11. Suppose that the population $P(t)$ of light blue Smurfs after t years is known to satisfy the differential equation $P'(t) = \frac{5}{P(t)}$.

a) Please derive the general formula for the number of light blue Smurfs present after t years, by the method of separation of variables.

b) How many light blue Smurfs are present after 30 years if the initial population has 10 Smurfs?

(20)   12. Please evaluate the following anti-derivaties, being sure to carefully show all work.

a) $\int 7x^4 \, dx$

b) $\int (e^x - \sin(3x)) \, dx$

c) $\int \frac{1}{2x} \, dx$

d) $\int 13 \csc(2x) \cot(2x) \, dx$

e) $\int (e^x + 2y) \, dy$

(20)   13. a) Please precisely state the Intermediate Value Theorem.

b) Please use the Intermediate Value Theorem to show that the function $f(x) = \tan(x) + \frac{1}{4}$ has a root on the interval $[-\frac{\pi}{4}, \frac{\pi}{4}]$.